Math 53 Discussion Problems Dec 5

1. Evaluate the following line/surface integrals by first converting them into a different form, using Stokes' theorem or divergence theorem.

   (a) $\int_C x^2dx + 2xdy + z^2dz$ where C is the ellipse $4x^2 + y^2 = 4$ in the xy-plane, oriented counterclockwise when viewed from above

   (b) $\iint_S (x^2\mathbf{i} + xz\mathbf{j} + 3z\mathbf{k}) \cdot d\mathbf{S}$ where S is the sphere $x^2 + y^2 + z^2 = 4$

   (c) $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$ where S is the oriented surface parametrized by $\mathbf{r}(r, \theta) = (r\cos\theta)\mathbf{i} + (r\sin\theta)\mathbf{j} + (9 - r^2)\mathbf{k}, 0 \leq r \leq 3, 0 \leq \theta \leq 2\pi$, $\mathbf{F} = (y - z)\mathbf{i} + (z - x)\mathbf{j} + (x + z)\mathbf{k}$

   (d) $\iint_S (x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}) \cdot d\mathbf{S}$ where S is the portion of the cylinder $x^2 + y^2 = 4$ between the planes $z = 0, z = 1$, oriented outwards

2. Let C be a simple closed smooth curve in the plane $2x + 2y + z = 2$, oriented counterclockwise when viewed from the positive x-axis. Show that $\oint_C 2ydx + 3zdy - xdz$ depends only on the area of the region enclosed by C in the plane.

3. Among all rectangular solids defined by the inequalities $0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq 1$, find the one for which the total flux of $F = (-x^2 - 4xy)\mathbf{i} - 6yz\mathbf{j} + 12z\mathbf{k}$ outwards through the six sides is the greatest.